Känguru der Mathematik 2016
Level Student (from grade 11)
Österreich – 17.03.2016

- 3 Point Questions -

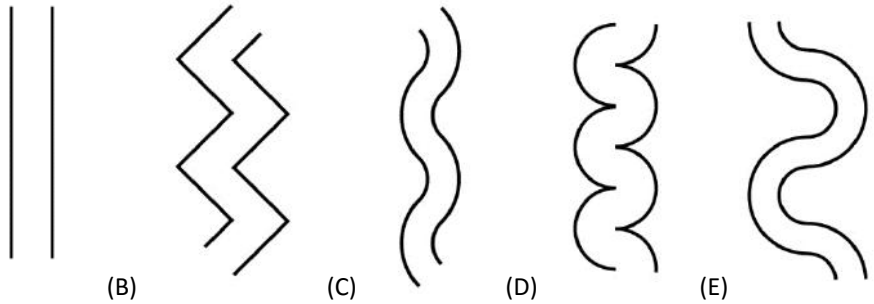
1. The sum of the ages of Tom and Johann is 23. The sum of the ages of Johann and Alex is 24 and the sum of the ages of Alex and Tom is 25. How old is the oldest of them?

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

2. The sum $\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}$ gives

(A) $\frac{3}{111}$ (B) $\frac{111}{1110}$ (C) $\frac{111}{1000}$ (D) $\frac{3}{1000}$ (E) $\frac{3}{1110}$

3. Maria wants to build a bridge across a river. This river has the special feature that from each point along one shore the shortest possible bridge to the other shore has always got the same length. Which of the following diagrams is definitely not a sketch of this river?

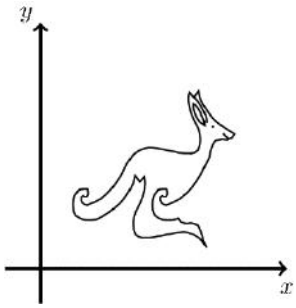


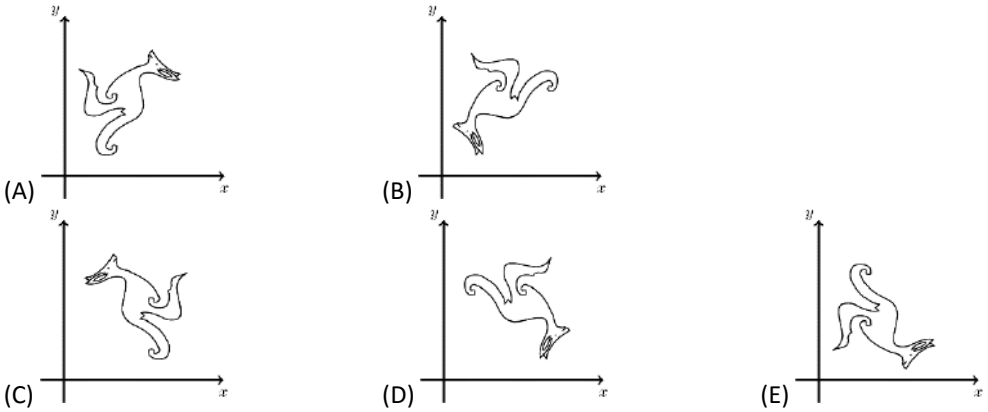
(A) (B) (C) (D) (E)

4. How many whole numbers are bigger than 2015×2017 but smaller than2016×2016?

(A) 0 (B) 1 (C) 2015 (D) 2016 (E) 2017

5. A scatter diagram on the *xy*-plane gives the picture of a kangaroo as shown on the right. Now the *x*- and the *y*-coordinate are swapped around for every point. What does the resulting picture look like?



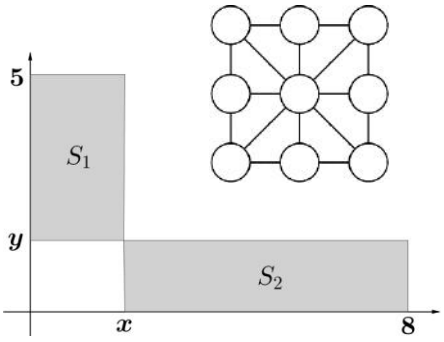


(A) (B) (C) (D) (E)

6. What is the minimum number of planes necessary to border a certain region in a three-dimensional space?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

7. Diana wants to write whole numbers into each circle in the diagram, so that for all eight small triangles the sum of the three numbers in the corners is always the same. What is the maximum amount of different numbers she can use?



(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

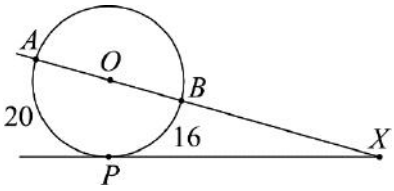
8. The rectangles S_1 and S_2 shown in the picture have the same area. Determine the ratio $x:y$.

(A) 1:1 (B) 3:2 (C) 4:3 (D) 7:4 (E) 8:5

9. If $x^2-4x+2=0$ then $x+\frac{2}{x}$ equals

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

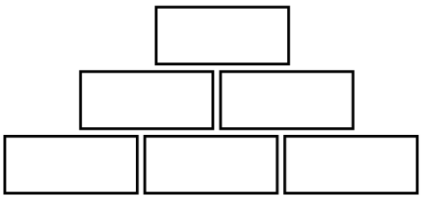
10. The diagram shows a circle with centre *O* as well as a tangent that touches the circle in point *P*. The arc *AP* has length 20, the arc *BP* has length 16. What is the size of the angle $\angle AXP$?



(A) 30° (B) 24° (C) 18° (D) 15° (E) 10°

- 4 Point Questions -

11. a, b, c, d are positive whole numbers for which $a + 2 = b - 2 = c \times 2 = d \div 2$ holds true.
Which of the four numbers a, b, c and d is biggest?



(A) a (B) b (C) c (D) d (E) It is not uniquely defined.

12. In this number pyramid each number in a higher cell is equal to the product of the two numbers in the cells immediately underneath that number. Which of the following numbers cannot appear in the topmost cell, if the cells on the bottom row hold natural numbers greater than 1 only?

(A) 56 (B) 84 (C) 90 (D) 105 (E) 220

13. Which value does x_4 take if $x_1 = 2$ and $x_{n+1} = {x_n}^{x_n}$ for $n \geq 1$?

(A) 2^{2^3} (B) 2^{2^4} (C) $2^{2^{11}}$ (D) $2^{2^{16}}$ (E) $2^{2^{768}}$

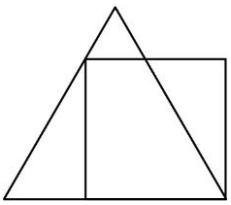
14. In rectangle $ABCD$ the side BC is exactly half as long as the diagonal $\overline{AC}$. Let X be the point on $\overline{CD}$ for which $|\overline{AX}| = |\overline{XC}|$ holds true. How big is the angle $\angle CAX$?

(A) 12.5° (B) 15° (C) 27.5° (D) 42.5° (E) another angle

15. Diana cuts a rectangle of area 2016 into 56 identical squares. The side lengths of the rectangle and the squares are all whole numbers. For how many different rectangles can she do this? (Two rectangles are said to be different if they are not congruent.)

(A) 2 (B) 4 (C) 6 (D) 8 (E) 0

16. The square shown in the diagram has a perimeter of 4. The perimeter of the equilateral triangle is



(A) 4 (B) $\mathbf{3} + \sqrt{\mathbf{3}}$ (C) 3 (D) $\mathbf{3} + \sqrt{\mathbf{2}}$ (E) $\mathbf{4} + \sqrt{\mathbf{3}}$

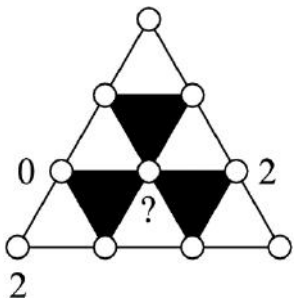
17. On the island of knights and liars everybody is either a knight (who only tells the truth) or a liar (who always lies). On your journey on the island you meet 7 people who are sitting in a circle around a bonfire. They all tell you "I am sitting between two liars!". How many liars are sitting around the bonfire?

(A) 3 (B) 4 (C) 5 (D) 6 (E) More information is necessary to make a decision.

18. Three three-digit numbers are built using the digits 1 to 9 so that each of the nine digits is used exactly once. Which of the following numbers cannot be the sum of the three numbers?

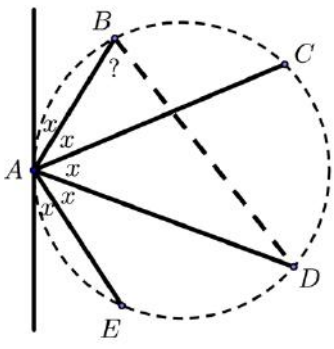
(A) 1500 (B) 1503 (C) 1512 (D) 1521 (E) 1575

19. Each of the ten points in the diagram is labelled with one of the numbers 0, 1 or 2. It is known that the sum of the numbers in the corner points of each white triangle is divisible by 3, while the sum of the numbers in the corner points of each black triangle is not divisible by 3. Three of the points are already labeled as shown in the diagram. With which numbers can the inner point be labeled?



(A) only 0 (B) only 1 (C) only 2 (D) only 0 and 1 (E) either 0 or 1 or 2

20. Bettina chooses five points A, B, C, D and E on a circle and draws the tangent to the circle at point A. She realizes that the five angles marked x are all equally big. (Note that the diagram is not drawn to scale!)
How big is the angle $\angle ABD$?



(A) 66° (B) 70.5° (C) 72° (D) 75° (E) 77.5°

- 5 Point Questions -

21. How many different real solutions does the following equation have?

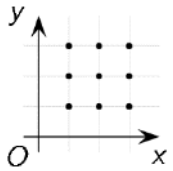
$$(x^2 - 4x + 5)^{x^2+x-30} = 1 \qquad ?$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

22. A quadrilateral has an inner circle (i.e. all four sides of the quadrilateral are tangents to the circle). The ratio of the perimeter of the quadrilateral to the circumference of the circle is 4:3. The ratio of the area of the quadrilateral to that of the circle is therefore

(A) $4:\pi$ (B) $3\sqrt{2}:\pi$ (C) $16:9$ (D) $\pi:3$ (E) $4:3$

23. How many quadratic functions $y = ax^2 + bx + c$ (with $a \neq 0$) have graphs that go through at least 3 of the marked points?



(A) 6 (B) 15 (C) 19 (D) 22 (E) 27

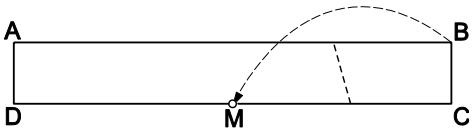
24. In the right-angled triangle ABC (with the right angle in A) the angle bisectors of the acute angles intersect at point P. The distance of P to the hypotenuse is $\sqrt{8}$. What is the distance of P to A?

(A) 8 (B) 3 (C) $\sqrt{10}$ (D) $\sqrt{12}$ (E) 4

25. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real solutions. It is known that the sum of the squares of the solutions of the first equation is equal to the sum of the squares of the solutions of the second equation and that $a \neq b$. $a + b$ equals

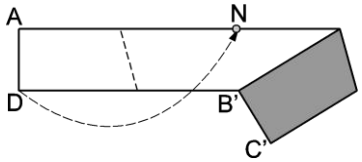
(A) 0 (B) -2 (C) 4 (D) -4 (E) The sum cannot be uniquely determined.

26. In a solid cube *P* is a point on the inside. We cut the cube into 6 (sloping) pyramids. Each pyramid has one face of the cube as its base and point *P* as its top. The volumes of five of these pyramids are 2, 5, 10, 11 and 14. What is the volume of the sixth pyramid?



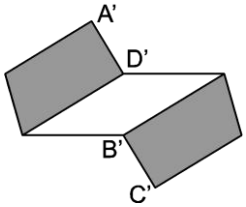
(A) 1 (B) 4 (C) 6 (D) 9 (E) 12

27. A rectangular piece of paper $ABCD$ is 5 cm wide and 50 cm long. The paper is white on one side and grey on the other. Christina folds the strip as shown so that the vertex B coincides with M the midpoint of the edge CD. Then she folds it so that the vertex D coincides with N the midpoint of the edge AB. How big is the area of the visible white part in the diagram?



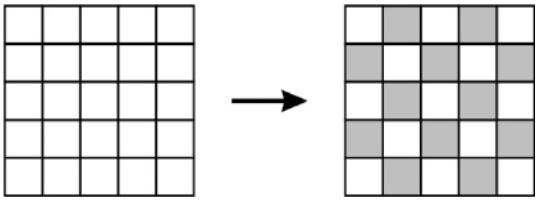
(A) 50 cm² (B) 60 cm² (C) 62.5 cm² (D) 100 cm² (E) 125 cm²

28. Anna chooses a positive whole number n and writes down the sum of all positive whole numbers from 1 to n. A prime number p divides this sum but none of the summands. Which of the following numbers is a possible value of $n + p$?



(A) 217 (B) 221 (C) 229 (D) 245 (E) 269

29. We consider a 5×5 square that is split up into 25 fields. Initially all fields are white. In each move it is allowed to change the colour of three fields that are adjacent in a horizontal or vertical line (i.e. white fields turn black and black ones turn white). What is the smallest number of moves needed to obtain the chessboard colouring shown in the diagram?



(A) less than 10 (B) 10 (C) 12 (D) more than 12 (E) This colouring cannot be obtained.

30. The positive whole number N has exactly six different (positive) factors including 1 and N. The product of five of these factors is 648. Which of these numbers is the sixth factor of N?

(A) 4 (B) 8 (C) 9 (D) 12 (E) 24