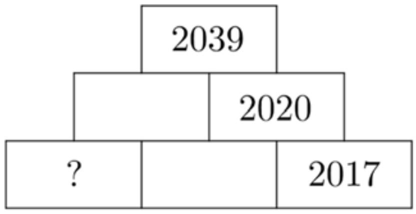
# Känguru der Mathematik 2017
# Level Student (Grade 11 to 13)
# Österreich - 16. 3. 2017

## - 3 Point Questions -

**1** On the number wall shown the number on each tile is equal to the sum of the numbers on the two tiles directly below it.
Which number is on the tile marked with "?" ?

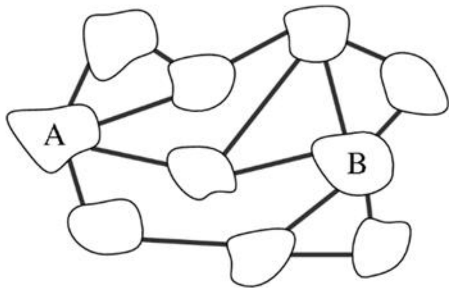



(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

**2** Many model railways use the H0-scale 1:87. For his railway Benjamin owns a 2 cm high model of his brother in H0-scale. How tall is his brother in reality?

(A) 1.74 m (B) 1.62 m (C) 1.86 m (D) 1.94 m (E) 1.70 m

**3** In the diagram we see 10 islands that are connected by 15 bridges. What is the minimum number of bridges that need to be closed off so that there is no connection from A to B anymore?

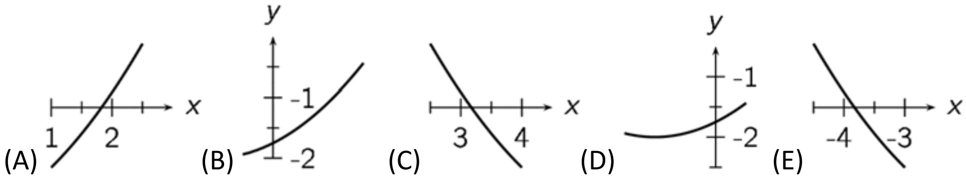



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

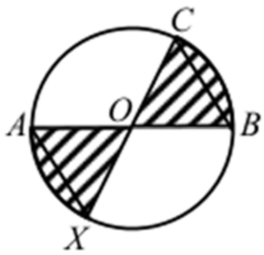
**4** Two positive numbers $a$ and $b$ have the following property: 75 % of $a$ is equal to 40 % of $b$. From that follows:

(A) *15a = 8b* (B) *7a = 8b* (C) *3a = 2b* (D) *5a = 12b* (E) *8a = 15b*

**5** Four of the following five pictures show pieces of the graph of the same quadratic function. Which piece does not belong?

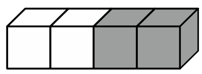


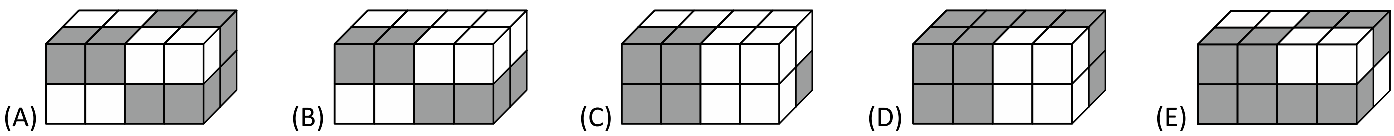

**6** The diagram shows a circle with centre $O$ and the diameters $AB$ and $CX$. Let $OB = BC$. Which fraction of the circle area is shaded?



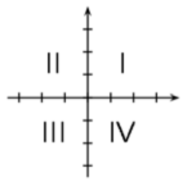

(A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{7}$ (D) $\frac{3}{8}$ (E) $\frac{4}{11}$

**7** A $4 \times 1 \times 1$ cuboid is made up of 2 white and 2 grey cubes as shown. Which of the following cuboids can be build entirely out of such $4 \times 1 \times 1$ cuboids?






**8** Which quadrant contains no points of the graph of the linear function $f(x) = -3.5x + 7$?




(A) I (B) II (C) III (D) IV (E) Every quadrant contains at least one point of the graph.

**9** In each of the five boxes (A) to (E) there are red and blue balls. Benedict wants to take exactly one ball without looking, out of one of these boxes and hopes to get a blue ball. In which box is the probability of that happening greatest?

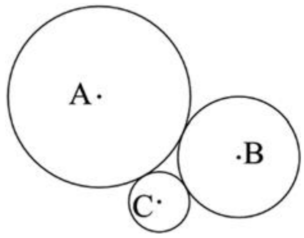
(A) 10 blues, 8 reds (B) 6 blues, 4 reds (C) 8 blues, 6 reds (D) 7 blues, 7 reds (E) 12 blues, 9 reds

**10** The graph of which of the following functions has the most intersections with the graph of the function $f(x) = x$ ?

(A) $g_1(x) = x^2$ (B) $g_2(x) = x^3$ (C) $g_3(x) = x^4$ (D) $g_4(x) = -x^4$ (E) $g_5(x) = -x$

## - 4 Point Questions -

**11** Three circles with centres $A$, $B$, $C$ touch each other in pairs from the outside (see diagram). Their radii are 3, 2 and 1.
How big is the area of the triangle $ABC$?



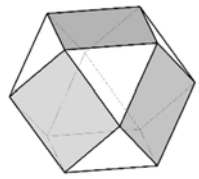

(A) 6 (B) $4\sqrt{3}$ (C) $3\sqrt{2}$ (D) 9 (E) $2\sqrt{6}$

**12** The positive number $p$ is smaller than 1, and the number $q$ is greater than 1.
Which of the following numbers is biggest?

(A) $p \times q$ (B) $p + q$ (C) $\frac{p}{q}$ (D) $p$ (E) $q$

**13** Two cylinders $A$ and $B$ have the same volume. The radius of the base of $B$ is 10 % bigger than that of $A$. How much is the height of $A$ greater than that of $B$?
(A) 5 % (B) 10 % (C) 11 % (D) 20 % (E) 21 %

**14** Each face of the polyhedron shown is either a triangle or a square. Each square borders 4 triangles, and each triangle borders 3 squares. The polyhedron has 6 squares. How many triangles does it have?



(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

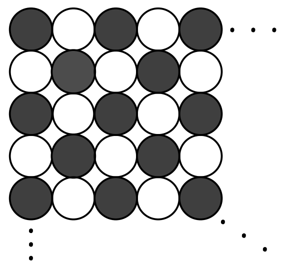
**15** The four faces of a regular tetrahedron are labelled with the four digits 2, 0, 1 and 7 (one digit on each face). For a game, four such tetrahedrons are used as fair dice. All four dice are thrown simultaneously. Three of the four faces of each die can then be seen from above.
What is the probability that we can form the number 2017 using exactly one of the three visible digits of each die?

(A) $\frac{1}{256}$ (B) $\frac{63}{64}$ (C) $\frac{81}{256}$ (D) $\frac{3}{32}$ (E) $\frac{29}{32}$

**16** The polynomial $5x^3 + ax^2 + bx + 24$ has whole number coefficients $a$ and $b$.
Which of the following numbers is definitely not a solution to the equation $5x^3 + ax^2 + bx + 24 = 0$?

(A) 1 (B) -1 (C) 3 (D) 5 (E) 6

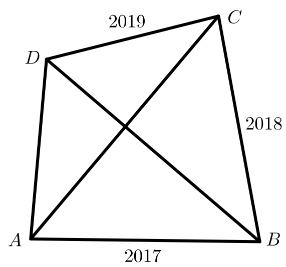
**17** Julia has 2017 round discs available: 1009 black ones and 1008 white ones. Using them, she wants to lay the biggest square pattern (as shown) possible and starts by using a black disc in the left upper corner. Subsequently she lays the discs in such a way that the colours alternate in each row and column. How many discs are left over when she has laid the biggest square possible?



(A) none (B) 40 of each colour (C) 40 black and 41 white ones
(D) 41 of each colour (E) 40 white and 41 black ones

**18** Two consecutive positive whole numbers are written on a board. The sum of the digits of each number is divisible by 7. What is the minimum number of digits the smaller of the two numbers has to have?
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

**19** In a convex quadrilateral $ABCD$ the diagonals are perpendicular to each other. The length of the edges are $AB = 2017$, $BC = 2018$ and $CD = 2019$ (diagram not to scale). How long is side $AD$?




(A) 2016 (B) 2018 (C) $\sqrt{2020^2 - 4}$ (D) $\sqrt{2018^2 + 2}$ (E) 2020

**20** Lilli tries to be a well-behaved kangaroo but she is having just too much fun not to lie every now and then. Therefore every third statement of hers is a lie and the rest is true. Sometimes she starts with a lie and sometimes with one or two true statements. Lilli thinks of a two-digit number and says to her friend:

1: "One digit of the number is a 2." 2: "The number is greater than 50." 3: "It is an even number."
4: "The number is less than 30." 5: "The number is divisible by 3." 6: "One digit of the number is a 7."

How big is the sum of the digits of the number, Lilli is thinking of?

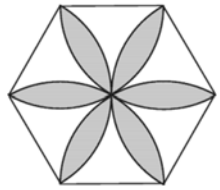
(A) 9 (B) 12 (C) 13 (D) 15 (E) 17

## - 5 Point Questions -

**21** How many positive whole numbers have the property that, if you delete the last digit you obtain a new number, which is exactly equal to 1/14 of the original number?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

**22** The diagram shows a regular hexagon with side length 1. The grey flower is outlined by circular arcs with radius 1 whose centre's lie in the vertices of the hexagon.
How big is the area of the grey flower?

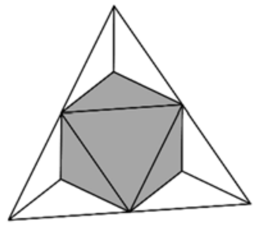


(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $2\sqrt{3}-\pi$ (D) $\frac{\pi}{2}+\sqrt{3}$ (E) $2\pi-3\sqrt{3}$

**23** We look at the sequence $\langle a_n \rangle$ with $a_1 = 2017$ and $a_{n+1} = \dfrac{a_n - 1}{a_n}$. Then: $a_{999} =$

(A) $-2017$ (B) $2017$ (C) $\frac{2016}{2017}$ (D) $1$ (E) $-\frac{1}{2016}$

**24** We look at a regular tetrahedron with volume 1. Its four vertices are cut off by planes that go through the midpoints of the respective edges (see diagram).
How big is the volume of the remaining solid?



(A) $\frac{4}{5}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

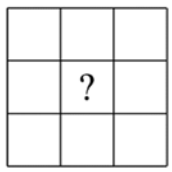
**25** The sum of the three side lengths of a right-angled triangle equals 18. The sum of the squares of these three side lengths equals 128. How big is the area of the triangle?

(A) 18 (B) 16 (C) 12 (D) 10 (E) 9

**26** Anna has five boxes, as well as five black balls and five white balls. She is allowed to decide how she shares out the balls between the boxes as long as she puts at least one ball into each box. Beate randomly chooses one box and takes one ball without looking. Beate wins if she draws a white ball. Otherwise Anna wins.
How should Anna distribute the balls in order to get the highest probability of winning?

(A) Anna puts one white and one black ball into each box.
(B) Anna distributes the black balls into three of the boxes and the white ones into the remaining two boxes.
(C) Anna distributes the black balls into four of the boxes and puts all of the white ones into the remaining box.
(D) Anna puts all of the white balls into one box and then puts one black ball into each box.
(E) Anna puts all of the black balls into one box and then puts one white ball into each box.

**27** Nine whole numbers were written into the cells of a $3 \times 3$-table. The sum of these nine numbers is 500. We know that the numbers in two adjacent cells (with a common sideline) differ by exactly 1.
Which number is in the middle cell?




(A) 50 (B) 54 (C) 55 (D) 56 (E) 57

**28** How big is $x+y$, if $|x|+x+y=5$ as well as $x+|y|-y=10$ holds true?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**29** How many different three-digit numbers $ABC$ are there so that $(A+B)^C$ is a three-digit power of two?

(A) 15 (B) 16 (C) 18 (D) 20 (E) 21

**30** 2017 people live on an island. Each person is either a liar (who always lies) or a nobleman (who always tells the truth). Over a thousand of them attend a banquet where they all sit together around one big round table. Everyone is saying, "Of my two neighbours, one is a liar and one is a nobleman." What is the maximum number of noblemen on the island?

(A) 1683 (B) 668 (C) 670 (D) 1344 (E) 1343