

# Känguru der Mathematik 2018

## Level Student (Grade 11 onwards)

### Austria - 15. 3. 2018



#### - 3 Points Examples -

1. In the diagram you can see the calendar page of a certain month. Unfortunately ink has run across parts of the page. Which day of the week does the 27th of that month fall on?

- (A) Monday (B) Wednesday (C) Thursday (D) Saturday (E) Sunday

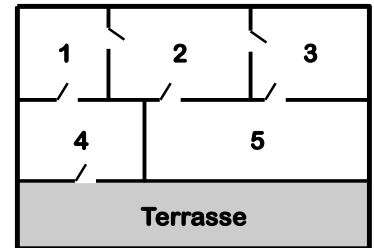


2. Which of the following expressions has the biggest value?

- (A)  $2 - 0 \cdot 1 + 8$  (B)  $2 + 0 \cdot 1 \cdot 8$  (C)  $2 \cdot 0 + 1 \cdot 8$  (D)  $2 \cdot (0 + 1 + 8)$  (E)  $2 \cdot 0 + 1 + 8$

3. The diagram shows the floor plan of Renate's house. Renate enters her house from the terrace (Terrasse) and walks through every door of the house exactly once. Which room does she end up in?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

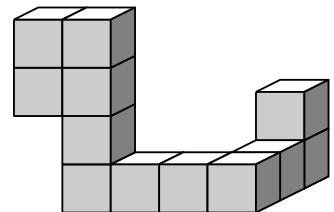


4. Thor has seven stones and a hammer. With his hammer he hits a stone and it breaks into five small stones. He does that a few times. Which of these numbers could be the number of stones he ends up with?

- (A) 17 (B) 20 (C) 21 (D) 23 (E) 25

5. The diagram shows an object made up of 12 dice glued-together. The object is dipped into some colour so that the entire outside is coloured in this new colour. How many of the small dice will have exactly four faces coloured in?

- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

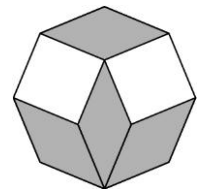


6. The following two statements are true: Some aliens are green and all others are purple. Green aliens live on Mars only. Which one of the following logical conclusions can be made?

- (A) All aliens live on Mars. (B) There are only green aliens on Mars.  
 (C) Some purple aliens live on Venus (D) All purple aliens live on Venus.  
 (E) There are no green aliens on Venus.

7. Four identical rhombuses (diamonds) and two squares are fitted together to form a regular octagon as shown. How big are the obtuse interior angles in the rhombuses?

- (A)  $135^\circ$  (B)  $140^\circ$  (C)  $144^\circ$  (D)  $145^\circ$  (E)  $150^\circ$

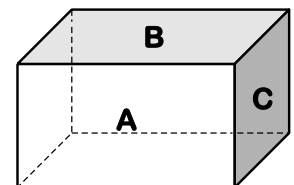


8. There are 65 balls in a box, 8 of which are white, the rest are black. Up to 5 balls can be taken out of the box in one draw. It is not allowed to put any balls back into the box. What is the minimum number of draws which have to be made to be certain that at least one white ball is drawn from the box?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

9. The faces of the brick have the areas A, B and C as shown. How big is the volume of the brick?

- (A)  $ABC$  (B)  $\sqrt{ABC}$  (C)  $\sqrt{AB + BC + CA}$  (D)  $\sqrt[3]{ABC}$  (E)  $2(A + B + C)$



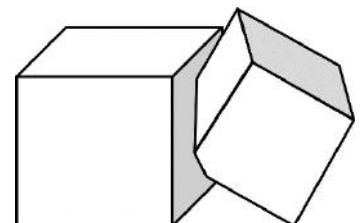
10. How many ways are there to write the number 1001 as the sum of two prime numbers?

- (A) no way (B) one way (C) two ways (D) three ways (E) more than three ways

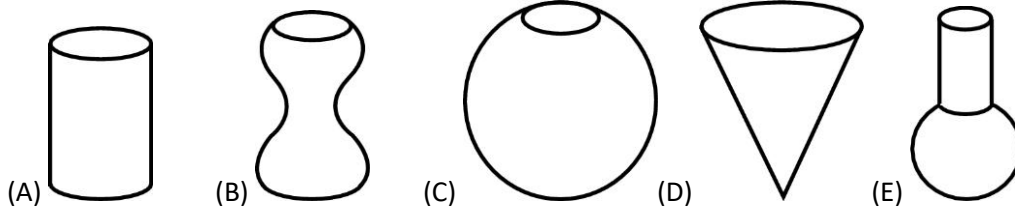
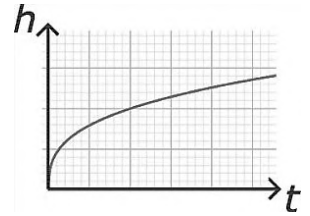
#### - 4 Point Examples -

11. Two dice with volumes  $V$  and  $W$  intersect each other as shown. 90% of the volume of the die with volume  $V$  does not belong to both dice. 85% of the volume of the die with volume  $W$  does not belong to both dice. What is the relationship between the volumes of the two dice?

- (A)  $V = \frac{2}{3} W$  (B)  $V = \frac{3}{2} W$  (C)  $V = \frac{85}{90} W$  (D)  $V = \frac{90}{85} W$  (E)  $V = W$



12. The five vases shown are filled with water. The filling rate is constant. For which of the five vases does the graph shown describe the height of the water  $h$  as a function of the time  $t$ ?

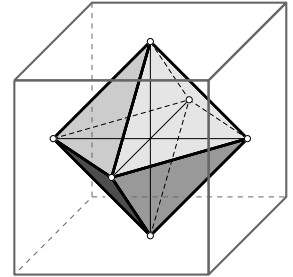


13.  $|\sqrt{17} - 5| + |\sqrt{17} + 5| =$

- (A) 10 (B)  $2\sqrt{17}$  (C)  $\sqrt{34} - 10$  (D)  $10 - \sqrt{34}$  (E) 0

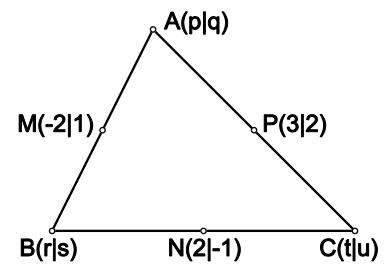
14. An octahedron is inscribed into a die with side length 1. The vertices of the octahedron are the midpoints of the faces of the die. How big is the volume of the octahedron?

- (A)  $\frac{1}{3}$  (B)  $\frac{1}{4}$  (C)  $\frac{1}{5}$  (D)  $\frac{1}{6}$  (E)  $\frac{1}{8}$



15. The vertices of a triangle have the co-ordinates  $A(p|q)$ ,  $B(r|s)$  and  $C(t|u)$  as shown. The midpoints of the sides of the triangle are the points  $M(-2|1)$ ,  $N(2|-1)$  and  $P(3|2)$ . Determine the value of the expression  $p + q + r + s + t + u$

- (A) 2 (B)  $\frac{5}{2}$  (C) 3 (D) 5 (E) another value



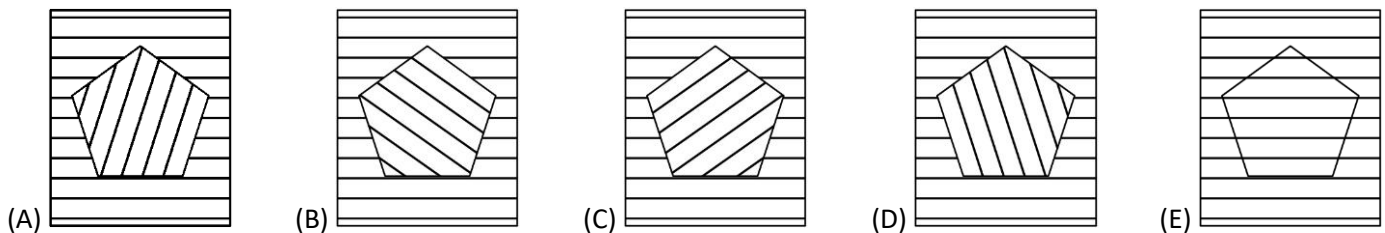
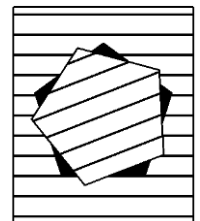
16. Before the football game, Real Madrid vs. Manchester United, the following five predictions were made:

- i) The game will not end in a draw.
- ii) Real Madrid will score at least one goal.
- iii) Real Madrid will not lose.
- iv) Real Madrid will win.
- v) Exactly three goals will be scored.

It turns out that exactly three of these predictions then come true. How many goals did Real Madrid score?

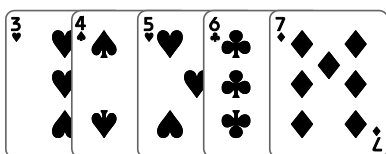
- (A) 0 (B) 1 (C) 2 (D) 3 (E) This cannot be determined for certain.

17. A regular pentagon is cut out of a page of lined paper. Step by step this pentagon is then rotated  $21^\circ$  counter clockwise about its midpoint. The result after step one is shown in the diagram. Which of the diagrams shows the situation when the pentagon fills the hole entirely again for the first time?



18. Which of the following numbers is not a factor of  $18^{2017} + 18^{2018}$ ?

- (A) 8 (B) 18 (C) 28 (D) 38 (E) 48

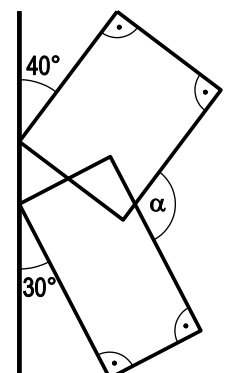


19. Three of the cards shown will be dealt to Nadia, the rest to Riny. Nadia multiplies the three values of her cards and Riny multiplies the two values of his cards. It turns out that the sum of those two products is a prime number. Determine the sum of the values of Nadia's cards.

- (A) 12 (B) 13 (C) 15 (D) 17 (E) 18

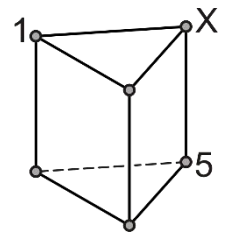
20. Two rectangles form the angles  $40^\circ$  and  $30^\circ$  respectively, with a straight line (see diagram). How big is angle  $\alpha$ ?

- (A)  $105^\circ$  (B)  $120^\circ$  (C)  $130^\circ$  (D)  $135^\circ$  (E) another value



**5 Point Examples**

**21.** The faces of the prism shown, are made up of two triangles and three squares. The six vertices are labelled using the numbers 1 to 6. The sum of the four numbers around each square is always the same. The numbers 1 and 5 are given in the diagram. Which number is written at vertex X?



- (A) 2      (B) 3      (C) 4      (D) 6      (E) This situation is impossible.

**22.**  $m$  and  $n$  are the solutions of the equation  $x^2 - x - 2018 = 0$ . What is the value of the expression  $n^2 + m$ ?

- (A) 2016      (B) 2017      (C) 2018      (D) 2019      (E) 2020

**23.** Four brothers with the harmonious names  $A$ ,  $B$ ,  $C$  and  $D$  are all of different heights. They make the following claims:

- $A$ : I am neither the tallest nor the smallest.       $B$ : I am not the smallest.  
 $C$ : I am the tallest.       $D$ : I am the smallest.

Exactly one of them lies. Who is the tallest brother?

- (A)  $A$     (B)  $B$     (C)  $C$     (D)  $D$     (E) Not enough information is given to be able to make a definite decision.

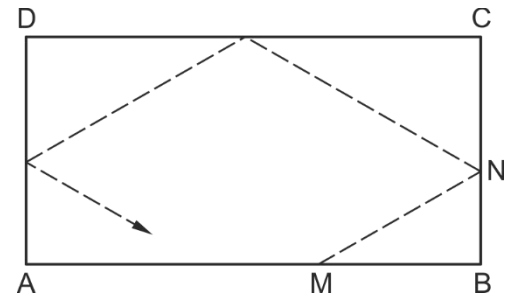
**24.** A function  $f$  fulfills the property  $f(x + y) = f(x) \cdot f(y)$  for all whole numbers  $x$  and  $y$ . Furthermore  $f(1) = 1/2$ . Determine the value of the expression  $f(0) + f(1) + f(2) + f(3)$ .

- (A)  $1/8$       (B)  $3/2$       (C)  $5/2$       (D)  $15/8$       (E) 6

**25.** A quadratic function of the form  $f(x) = x^2 + px + q$  intersects the  $x$ -axis and the  $y$ -axis in three different points. The circle through these three points intersects the graph of the function  $f$  in a fourth point. What are the coordinates of this fourth point of intersection?

- (A)  $(0 | -q)$     (B)  $(p | q)$     (C)  $(-p | q)$     (D)  $(-\frac{q}{p} | \frac{q^2}{p^2})$     (E)  $(1 | p + q + 1)$

**26.** On an idealised rectangular billiard table with side lengths 3 m and 2 m a ball (point-shaped) is pushed away from point  $M$  on the long side  $AB$ . It is reflected exactly once on each of the other sides as shown. at which distance from the vertex  $A$  will the ball hit this side again if  $BM = 1,2$  m and  $BN = 0,8$  m?

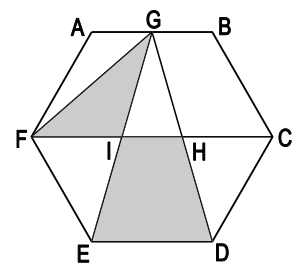


- (A) 2 m      (B) 1,5 m      (C) 1,2 m      (D) 2,8 m      (E) 1,8 m

**27.** How many real solutions does the equation  $||4^x - 3| - 2| = 1$  have?

- (A) 2      (B) 3      (C) 4      (D) 5      (E) 6

**28.**  $ABCDEF$  is a regular hexagon, as shown in the diagram.  $G$  is the midpoint of  $AB$ .  $H$  and  $I$  are the intercepts of the line segments  $GD$  and  $GE$  respectively, with the line segment  $FC$ . How big is the ratio of the areas of the triangle  $GIF$  and the trapezium  $IHDE$ ?



- (A)  $\frac{1}{2}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{4}$       (D)  $\frac{\sqrt{3}}{3}$       (E)  $\frac{\sqrt{3}}{4}$

**29.** In a class there are 40% more girls than boys. The probability that a student representative team of two students randomly selected from this class is made up of exactly one girl and one boy is exactly  $\frac{1}{2}$ . How many children are there in this class?

- (A) 20      (B) 24      (C) 36      (D) 38      (E) This situation is not possible.

**30.** Archimedes has calculated  $15!$ . The result is on the board.

Unfortunately two of the digits, the second and the tenth, cannot be read.

1 ■ 0767436 ■ 000

What are the two missing digits?

(Remark:  $15! = 15 \cdot 14 \cdot 13 \cdot \dots \cdot 2 \cdot 1$ )

- (A) 2 and 0      (B) 4 and 8      (C) 7 and 4      (D) 9 and 2      (E) 3 and 8