# Canguru de Matemática Brasil – LEVEL S – 2020 – Second Application

3 points

1. What is the last digit of the multiplication $1 \times 3 \times 5 \times 7 \times 9^2 \times 7 \times 5 \times 3 \times 1$ result?

(A) 1   (B) 3   (C) 5   (D) 7   (E) 9

2. An ant walked 6 m every day to go from point A to point B in a straight line. One day Johnny put a straight cylinder of one meter high in that way. Now the ant walks on the same straight line or above it, having to go up and down the cylinder, as shown in the picture. How much does she have to walk now to go from A to B?
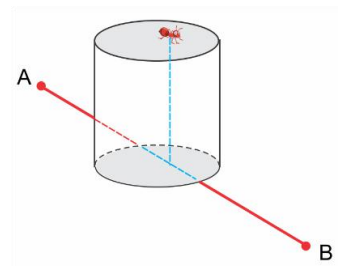
(A) 8 m   (B) 9 m   (C) $6 + \pi$ m   (D) $12 - \pi$ m   (E) 10 m

3. How many integers are there between $2020{,}9^2$ and $2018{,}9 \times 2022{,}9$?

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

4. What is the value of $\dfrac{1010^3 - 2020^3 + 3030^3}{1010^3}$?

(A) 10   (B) 20   (C) 30   (D) 40   (E) 50

5. The pie chart beside refers to the number of inhabitants of the five zones of a city. The central zone has the same population as the north, west and east zones together and the south zone has half of the inhabitants of the west zone. What is the percentage difference between the inhabitants of the north and east zones?
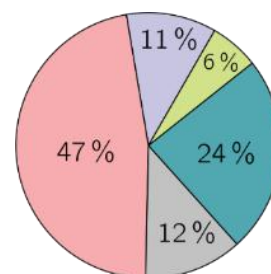
(A) 6%   (B) 11%   (C) 12%   (D) 13 %   (E) 18%

6. Being a, b e c integers numbers such that $1 \le a = b \le c$ and $abc = 2020^2$, what is the highest possible value of a?

(A) 36   (B) 40   (C) 44   (D) 101   (E) 202

7. Which of the following numbers is divisible by 3, whatever the integer n?

(A) $5n + 1$ (B) n^2 (C) $(n - 3)^2$ (D) $n^2 - 1$ (E) $n^3 - n$

8. If a dozen bananas nanicas cost the same as a dozen bananas prata and y bananas nanicas cost x reais, how many reais are z bananas prata?

(A) xyz (B) $\dfrac{5}{6}xyz$ (C) $\dfrac{6xz}{5y}$ (D) $\dfrac{5yz}{6x}$ (E) $\dfrac{6yz}{5x}$

9. Two equal dice have two red faces, two blue and two green each. If we roll the two dice simultaneously, what is the probability that the result will be two faces with different colors?

(A) $\frac{1}{12}$　　　　(B) $\frac{1}{9}$　　　　(C) $\frac{1}{6}$　　　　(D) $\frac{2}{9}$　　　　(E) $\frac{2}{3}$

10. In the addition on the right, different letters represent different numbers. Assuming the account is correct, what is the highest possible value for the sum C + A + N?

$$\begin{array}{r} C\ A\ N \\ +\ G\ U\ R \\ \hline U\ U\ U \end{array}$$

(A) 15　　　　(B) 18　　　　(C) 19　　　　(D) 21　　　　(E) 24

4 points

11. A gray square with an area of 36 cm² and a black square with an area of 25 cm² are superimposed, as shown beside. What is the perimeter of the overlapping region, represented by the white quadrilateral, which has a vertex on the side of the gray square?
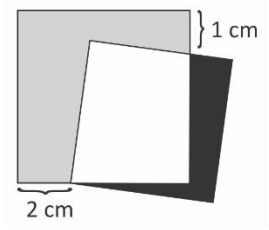

(A) It is not determined.　(B) 11 cm　　(C) 16 cm　　(D) 18 cm　　(E) 20 cm

12. Two thousand and twenty coins are on a table, with "head" facing up. With each movement you must turn exactly three of these coins. What is the smallest number of moves you must make so that all the coins on the table have "tales" facing up?

(A) 672　　　(B) 673　　　(C) 674　　　(D) 675
(E) It is not possible to make all the "tales" face up.

13. Zilda will use six equal cubes and two different rectangular blocks to form the structure beside with eight faces. Before gluing the pieces, she will paint each one entirely and calculated that she will need 18 liters of paint (the color does not matter). How many liters of paint would she use if she painted the whole structure only after gluing the parts?
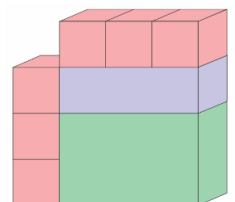
(A) 8,4　　　　(B) 9,6　　　　(C) 11,5　　　　(D) 12,8　　　　(E) 16,0

14. Let a, b and c be real numbers not null such that $(a - a^{-1})^2 + (b - b^{-1})^2 + (c - c^{-1})^2 = 0$.
What number below can **NOT** be the value of $a + b + c$?

(A) -3　　　　(B) -1　　　　(C) 0　　　　(D) 1　　　　(E) 3

15. The last two digits of a 2020 number are 9 and 9. At most, how many digits does the square of that number have?


(A) 2018　　　　(B) 2020　　　　(C) 4018　　　　(D) 4019　　　　(E) 4040

16. The sequence f_n is given by $f_1 = 1, f_2 = 2$ e $f_n = f_{n-1} \cdot f_{n+1}$ for $n \geq 2$. How many of the first 2020 elements of this sequence are even numbers?

(A) 673   (B) 674   (C) 1 010   (D) 1 346   (E) 1 347

17. Matias wrote 15 numbers on the wheel represented beside. Only one of them is visible, the 10 on top of the wheel. The sum of the numbers in any seven consecutive positions, such as the gray positions in the figure, does not vary. When seven numbers in consecutive positions are summed up, which of the following results is possible?
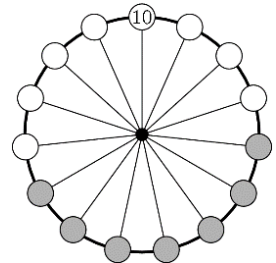

(A) 49   (B) 70   (C) 75   (D) 105   (E) 150

18. A large square touches another two squares, as shown in the picture. The numbers inside the smaller squares indicate their areas. What is the area of the largest square?
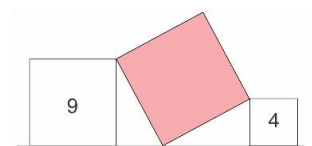

(A) 10   (B) 11   (C) 12   (D) 13   (E) 14

19. A circle is tangent to one side of a rectangle and passes through two of its vertices, as shown beside. A square of 20 cm² area has one side over the side of the rectangle and two vertices over the circle, as shown in the figure. What is the area of the rectangle?
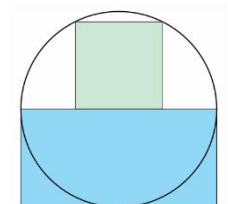
(A) 40 cm²   (B)45 cm²   (C) 50 cm²   (D) 55 cm²   (E) 60 cm²

20. Two rectangular blocks and a cube are joined to form a larger rectangular block, which volume is 280 cm³ . The cube, in dark gray in the picture, has volume equal to 125 cm³ and the smaller rectangular block has volume equal to 75 cm³ . What is the area of the face marked with the question mark?
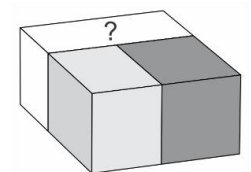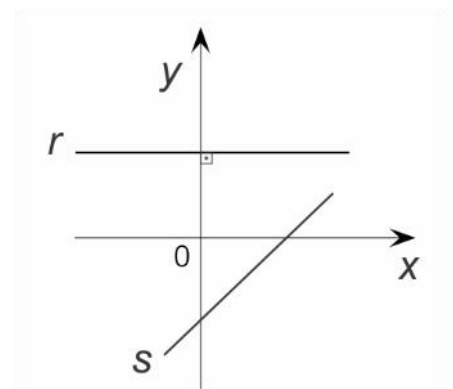

(A) 16 cm²   (B) 18 cm²   (C) 20 cm²   (D) 24 cm²   (E) 56 cm²

5 points

21. The figure shows the lines r and s, which equations are, respectively, $y = ax + b$ e $y = cx + d$. Which of the following statements is true?

(A) $ab + cd < 0$ (B) $a + b + c + d < 0$ (C) $ac + bd \geq 0$
(D) $a + b + c + d > 0$ (E) $abcd > 0$

22. A little kangaroo draws a line passing through point P of the grid and then paints three triangles in black as shown in the picture. The areas of these triangles are proportional to which numbers?

(A) 1 : 4 : 9   (B) 1 : 2 : 9   (C) 1 : 3 : 9   (D) 1 : 2 : 3   (E) 2 : 3 : 4
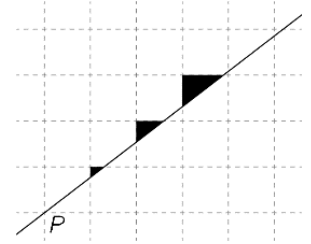
23. A rectangular garden was 50 m long and 40 m wide. An artificial lake was built next to it, so that the whole set forms a 60 m square. Then a fence was stretched, separating both the garden and the lake in two parts with equal areas, as shown in the picture. How long is this fence?

(A) 60 m   (B) $30\sqrt{5}$ m   (C) $60\sqrt{2}$ m   (D) 85 m   (E) $60\sqrt{3}$ m
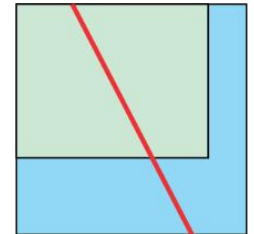
24. A positive integer N is divisible by all integers from 2 to 11 except two of these numbers. Among the pairs of integers (6,7), (7,8), (8,9), (9,10) e (10,11), how many could be this exception?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

25. At the Sunday fair, in the morning Ana wanted to buy three types of fruit among 12 options and one type of vegetable, among the six types available. In the afternoon some products were sold out and Bela wanted to buy two kinds of fruits and two kinds of vegetables, among the remaining ones. Since the number of possible choices for Bela was a quarter of the number of possible choices for Ana, how many products were sold out in the afternoon?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

26. Vilma took a sheet of paper measuring 10 cm x 20 cm and made two folds, taking the two smaller sides of the sheet to a diagonal of it. She gets a parallelogram, as shown in the picture. What is the area of this quadrilateral, in cm²?
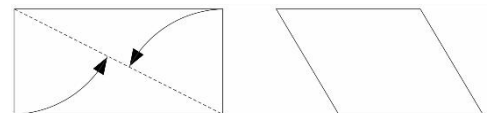
(A) $\dfrac{100\sqrt{5}}{3}$ (B) $50\sqrt{5}$ (C) $100(\sqrt{5}-1)$ (D) $50(5-\sqrt{5})$ (E) $50(5+\sqrt{5})$

27. The submerged volume of an iceberg in the form of a cube corresponds to 96,4% of the volume of the iceberg. If the outside of the water has the same three edges, what is the percentage of the surface area in contact with the air in relation to the total surface area of the iceberg?

(A) 9%   (B) 12%   (C) 15%   (D) 18%   (E) 21%

28. Maria writes all the positive divisors of 2020, one on each card and puts all these cards in a box. Then she closes her eyes and starts taking these cards out of the box, one by one. How many cards must she take out of the box to make sure that among the cards taken there are two with numbers a and b such that a is not a divisor of b and b is not a divisor of a?
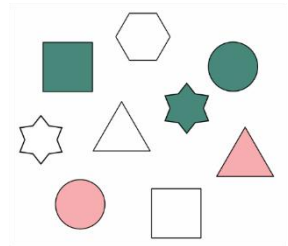
(A) 5   (B) 6   (C) 7   (D) 8   (E) 9

29. There are n different prime numbers $p_1, p_2, \dots, p_n$ written from left to right on the last line below the table shown beside. The product of two neighboring numbers in the same line is written in the upper two boxes. The number $K = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots p_n^{\alpha_n}$ is written in the last house above. In a table like this, in which $\alpha_2 = 9$, how many numbers are divisible by number p_4?
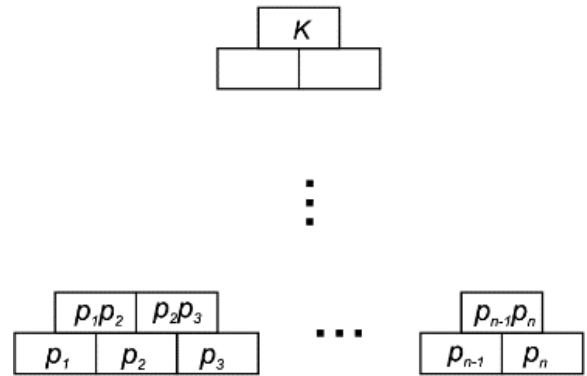
(A) 4   (B) 16   (C) 24   (D) 28   (E) 36

30. Adam and Bruna try to find out which is Carla's favorite figure, amongst the figures beside. Adam knows that Carla told Bruna what the shape of the figure was. Bruna knows that Carla told Adam what color the figure was. The following conversation takes place. Adam: "I don't know what Carla's favorite figure is and I know that Bruna doesn't know either". Bruna: "At first I didn't know what Carla's favorite figure was, but now I know". Adam: "Now I know too". What is Carla's favorite figure?

(A) ⬤   (B) ⬛   (C) ✦   (D) ⬜   (E) ▲