

## Canguru de Matemática Brasil – Level J – 2020 – Second Application

### 3 points

1. When the results of the following additions are written in descending order, what sum will be in the middle?

- (A)  $1 + 2 + 345$     (B)  $12 + 3 + 45$     (C)  $1 + 23 + 45$     (D)  $12 + 34 + 5$     (E)  $1 + 234 + 5$

2. Who is the mother of the daughter of the mother of Lia's daughter?

- (A) Lia's sister.    (B) Lia.    (C) Lia's mother.    (D) Lia's niece.    (E) Lia's aunt.

3. When Cosme correctly wears his new shirt, as shown on the left figure, the horizontal stripes form seven closed arches around his body. This morning he buttoned his shirt in the wrong way, as shown on the right. How many open arches were there around Cosme's body this morning?



- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

4. Four integer numbers have sum  $S$  and product 9. What is the lowest possible value for  $S$ ?

- (A)  $-18$     (B)  $-12$     (C)  $-8$     (D) 0    (E) 6

5. Maria has ten pieces of paper. Some are square and the others are triangles. She cuts each square along one of its diagonals. She then counts the total number of vertices of the pieces she has now and gets 48. How many squares did she have before making the cuts?

- (A) 3    (B) 4    (C) 5    (D) 6    (E) 7

6. In the addition beside, different letters represent different numbers and equal letters represent equal numbers. The resulting sum is a number of four digits, B being different from zero. What is the sum of the numbers of this number?

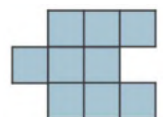
$$\begin{array}{r}
 A \\
 + AB \\
 ABA \\
 \hline
 BEBA
 \end{array}$$

- (A) AA    (B) BB    (C) AB    (D) BE    (E) EA

7. The figures for the year 2020 are repeated twice each and are different. How many times does this happen since the year 1000 until now?

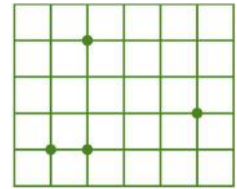
- (A) 11    (B) 17    (C) 20    (D) 24    (E) 29

8. There are several figures that can be formed by nine squares of 1 cm side by side (see an example beside) and one of them has the biggest perimeter. What is this perimeter?



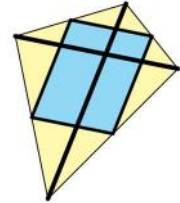
- (A) 12 cm    (B) 14 cm    (C) 16 cm    (D) 18 cm    (E) 20 cm

9. Four points were marked on a grid of 1 cm side squares. Of the possible triangular regions that can be obtained with vertices in three of these points, one has the largest area. What is this area, in  $\text{cm}^2$ ?



- (A) 2,5      (B) 3,5      (C) 4,5      (D) 5,5      (E) 6,5

10. Martinho made a bicolored kite with six pieces of a thin strip of bamboo. Two pieces were used for the diagonals, which are perpendicular. The other four pieces were used to connect the middle points on the sides of the kite, as shown in the picture. What is the ratio between the blue and yellow parts of the kite?



- A)  $\frac{2}{3}$       (B)  $\frac{3}{2}$       (C)  $\frac{3}{4}$       (D)  $\frac{4}{3}$       (E) 1

**4 points**

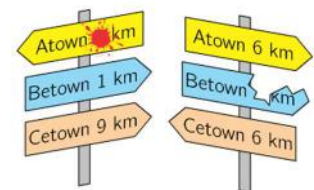
11. Adam, Breno and Carlos live in the same apartment and bought a treadmill for exercise. Nobody uses the treadmill on Wednesdays and Sundays and there is no day the three of them use the treadmill. Adam uses the treadmill 4 times a week and Breno 5 times a week. There are 4 consecutive days when Adam uses the treadmill on the first day, he does not use it on the second day and Breno does not use it on the fourth day. Carlos uses the treadmill on one day of the week. Which day?

- (A) Friday      (B) Saturday      (C) Tuesday      (D) Thursday      (E) Monday

12. Integers  $a$ ,  $b$ ,  $c$  and  $d$  satisfy the equality  $2ab = 3cd$ . Which of the following numbers can be the  $abcd$  product?

- (A) 50      (B) 100      (C) 150      (D) 200      (E) 250

13. The shortest way from Atown to Cetown is through Betown. Going back by this road from Cetown to Atown, we first find the signposts on the left side of the road. Further on we find the road signs on the right side of the road. How far is it from Betown to Atown?

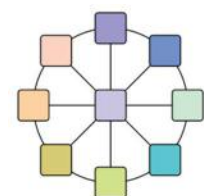


- (A) 1 km      (B) 2 km      (C) 3 km      (D) 4 km      (E) 5 km

14. Two isosceles triangles not similar have at least one side of 20 cm and have equal perimeters. If one of them has a side of 8 cm, which of the following measures can be the measure of one side of the other triangle?

- (A) 8 cm      (B) 10 cm      (C) 12 cm      (D) 14 cm      (E) 16 cm

15. Toninho wants to write strictly positive and consecutive whole numbers, in the nine places of the figure, so that the sum of the three numbers in each diameter is equal to 24. What is the largest possible sum for all the nine numbers?

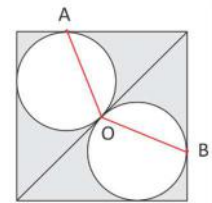


- (A) 81      (B) 90      (C) 99      (D) 108      (E) 117

16. Marta observed that the number 2020 has the following property: the number formed by the two digits on the left is equal to the number formed by the two digits on the right. How many four-digit numbers, including 2020, have this same property?

- (A) 49                      (B) 50                      (C) 81                      (D) 90                      (E) 99

17. Two circles are tangent to each other and also to two sides of a square. What is the measure of the  $A\hat{O}B$  angle, determined by three of these points of tangency, as shown in the figure?



- (A)  $110^\circ$                       (B)  $112^\circ$                       (C)  $120^\circ$                       (D)  $128^\circ$                       (E)  $135^\circ$

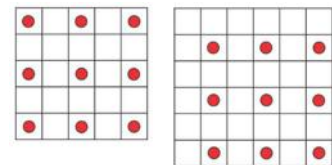
18. Roberto and Maria leave at the same time from the same point of a long circular track, he on foot and she by bike. Maria completes a lap 24 minutes before Roberto and waits for him while having an ice cream. When he reaches this point, Maria leaves with her bike in the opposite direction and Roberto continues walking without stopping in the same direction. Then they meet 5 minutes later. Assuming the speeds are kept constant, how long does it take for Roberto to take a ride on the track?

- (A) 30 min                      (B) 29 min                      (C) 28 min                      (D) 27 min                      (E) 26 min

19. Let  $17x + 51y = 102$ . If  $x$  e  $y$  are strictly positive integers, what is the value of  $x - 3y$ ?

- (A)  $-3$                       (B) 0                      (C) 3                      (D) 12                      (E) undetermined

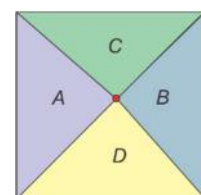
20. Ana plays with  $n \times n$  boards by placing a token in each of the cells with no common points with other cells containing tokens. In the picture beside we see how to place as many chips as possible on  $5 \times 5$  and  $6 \times 6$  boards. In this way, how many chips can Ana possibly put on a  $2020 \times 2020$  board?



- (A) 2020                      (B) 4039                      (C)  $674^2$                       (D)  $1010^2$                       (E)  $2020^2$

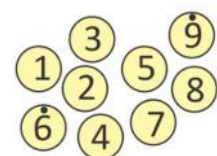
**5 points**

21. The next window is a square of area  $1 \text{ m}^2$  and is composed of four triangles, which areas, indicated in the figure, follow the ratios  $3A = 4B$  and  $2C = 3D$ . A fly is placed exactly at the point where these four triangles touch each other. The fly flies directly to the side closest to the window. How much does it fly?



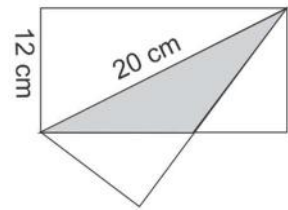
- (A) 40 cm                      (B) 30 cm                      (C) 25 cm                      (D) 20 cm                      (E) 10 cm

22. Julia puts the nine chips on the right in a box. She then takes one chip at a time, without looking, and notes down its digit, obtaining, at the end, a number of nine different digits. What is the probability that the number written by Julia is divisible by 45?



- (A)  $\frac{1}{9}$                       (B)  $\frac{2}{9}$                       (C)  $\frac{1}{3}$                       (D)  $\frac{4}{9}$                       (E)  $\frac{8}{9}$

23. A rectangular sheet with one side of 12 cm is folded along its 20 cm diagonal. What is the overlapping area of the folded parts, indicated in gray in the picture beside?

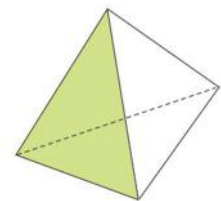


- (A)  $24 \text{ cm}^2$       (B)  $36 \text{ cm}^2$       (C)  $48 \text{ cm}^2$       (D)  $50 \text{ cm}^2$       (E)  $75 \text{ cm}^2$

24. Carlos always tells the truth on alternate days. On the other days, he tells only lies. Today he made exactly four of the five statements that follow. Which one was not made today by him?

- (A) The number of male and female friends I have is a prime number.  
 (B) The number of male friends I have is equal to the number of female friends I have.  
 (C) My name is Carlos.  
 (D) Three of my male friends are older than me.  
 (E) I always tell the truth.

25. Julia wrote four positive integers, one at each vertex of a triangular base pyramid. She calculated the sum of the numbers written on the vertices of one face and the product of the numbers written on the vertices of other two faces, obtaining 15, 20 and 30, respectively. What is the highest possible value of the product of the four numbers?



- (A) 40      (B) 50      (C) 60      (D) 90      (E) 120

26. Pedro assembled a cube using 64 little white equal cubes and then he painted the cube red. Then he dismantled the cube and reassembled it so that all its faces were white and painted the cube red again. How many white faces of little cubes remained white?

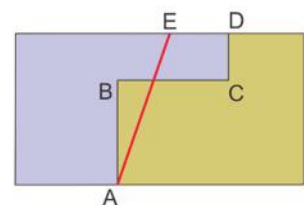
- (A) 0      (B) 72      (C) 109      (D) 144      (E) 192

27. Ana wants to write positive whole numbers, one in each of the squares beside, so that the sums of the four numbers in each row and the four numbers in each column are equal. She has already written some numbers, as shown. She wants to write the missing numbers so that the sum of these six numbers is as small as possible. What is this sum?

1		6	3
	2	2	8
	7		4
		7	

- (A) 32      (B) 24      (C) 20      (D) 18      (E) 12

28. A large rectangular plot is divided into two lots that are separated from each other by an ABCD fence, as shown in the picture beside. The AB, BC and CD parts of this fence are parallel to the sides of the rectangle and have lengths of 30 m, 24 m and 10 m, respectively. The owners of these lands have combined to knock down the fence and make a new straight AE fence, without changing the area of each of the lands. How far from point D should the E end of the fence be?



- (A) 8 m      (B) 10 m      (C) 12 m      (D) 14 m      (E) 16 m

29. The number  $K = 9999 \dots 9$  is formed by  $n$  digits 9. What is the sum of the digits of the number  $K^3$  ?

(A)  $n + 17$

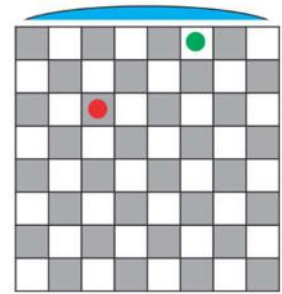
(B)  $\underbrace{999 \dots 9}_{3n \text{ 9's}}$

(C)  $\underbrace{999 \dots 9}_{n \text{ 9's}}$

(D)  $18n$

(E)  $10^{6n} - 1$

30. On the 8 x 8 board beside, in how many ways can you place two chips, one green and one red, in different colored cells, so that the chips are not in the same row or in the same column of the board?



(A) 112

(B) 672

(C) 768

(D) 1344

(E) 1536